

ON THE THREEDIMENSIONAL FINITE ELEMENT METHOD IN THE ENERGY EFFICIENCY OF BUILDING'S ENVELOPE

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Abstract. Contemporary design in the civil engineering becomes shorter in time, more efficient and less expensive by the use of numerical methods of computational mechanics [1]. Therefore, it is worthwhile to see how finite element method [1] is used in the calculation of heat gain/losses of non-homogenous building envelopes, in order to estimate energy savings achieved by the optimization of mass, price and quality of applied building envelope materials. The program package, with the one of the best cost/quality ratio on the market, named "Straus7" [2], as well as in-house software package "FEMIX HC : $u/\sigma - T/q$ " [3,4] is used in the calculations. Obtained results show that the use of the proposed program packages gives reliable results, with the possibility to optimize the rate: envelope price/ minimization of heat losses, either in design stage, either in the process of the revitalization of the building envelope, in order to increase energy efficiency of the civil facilities.

Key-words: building envelope, non-homogenous wall, heat transfer, mathematical model, numerical simulation, energy efficiency, finite element method, HVAC, U-value

1. Introduction

The purpose of this paper is to demonstrate the use of the finite element method in the 3-D problem of heat transfer. In addition, to point out that the use of this method leads to quick and inexpensive estimation of the energy efficiency/envelope price relationship. The design of the energy efficient building also implies its optimal positioning according to the local meteorological image, and after that the calculation of the heating or cooling load of the particular rooms, as to be in position to design and build HVAC system [5, 6], which is going to maintain designed comfort conditions. For this purpose we have calculation standards and methods, and mostly used are DIN 4701, DIN 1946, VDI 2078 i ASHRAE.

In addition, new simulation methods i.e. computational mechanics methods, which successfully deals with variety of non steady-state variables, such as sun radiation intensity, sky, ground and surrounding surfaces radiation, outdoor air temperature, wind velocity, number of persons and period of their lingering in the room, lightning and other internal heat sources power and regime of operation, are today frequently used [1]. These are based on the 3-D physical equations of non steady-state heat transfer of building surfaces and on a number of expressions which determine boundary conditions on the those surfaces, where the mechanisms of the heat transfer, such as conduction, convection, ambient and sun radiation, as well as radiation of the indoor elements, are analyzed [7]. One of the good qualities of the numerical simulation software packages is a graphical visualization of obtained results. This visualization ensures fast identification of the model zones in which local extremes of the analyzed variables fields occur. In this case, visualization may be per temperature and heat flux field.

In the present paper, heat losses of the building envelope due to heat conduction, convection and radiation, will be analyzed by the finite element method [1,4]. The advantage of the finite element method over other numerical methods, as for example finite volume method, is that it does not suffer if the geometry of the model is complex.

The purpose of this paper is to prove reliability and efficiency of the 3-D finite element method in analysis of the steady-state building envelope heat transfer, in which temperature and heat flow fields, i.e. thermal conductance – U value, are calculated. Beside that, the results of the verification of the analyzed numerical methods on the steady state model of the non-homogenous building envelope with heat bridges will be shown.

2. Finite element formulations in steady state heat transfer

The state of the body is described by the temperature T and the heat flux vector \mathbf{q} . Let us consider a complete system of field equations for steady-state heat transfer in the strong form, where:

$$\operatorname{div} \mathbf{q} + f = 0 \text{ in } \Omega, \quad (1)$$

$$\mathbf{q} = -\mathbf{k} \nabla T \text{ in } \Omega, \quad (2)$$

are respectively, the equation of thermal balance that states that the divergence of the heat flux is equal to the internal heat source f , and Fourier's law of heat conduction, which assumes that the heat flux is linearly related to the negative gradient of the temperature, where \mathbf{k} is second order tensor of thermal conductivity, which is heat transfer property of an general orthotropic material. If the material is homogeneous and isotropic, the tensor \mathbf{k} will degenerate to simple scalar value k , i.e. thermal conductivity coefficient. Nevertheless, the present approach considers full tensor of thermal conductivity.

These two equations are subjected to the following boundary conditions:

$$T = \bar{T} \text{ on } \partial\Omega_T, \quad (3)$$

$$\mathbf{q} \cdot \mathbf{n} = q_h = h \text{ on } \partial\Omega_q, \quad (4)$$

$$\mathbf{q} \cdot \mathbf{n} = q_c = h_c(T - T_a) \text{ on } \partial\Omega_c, \quad (5)$$

$$\mathbf{q} \cdot \mathbf{n} = q_r = h_r \sigma A(T^4 - T_a^4) \text{ on } \partial\Omega_r, \quad (6)$$

which are, boundary conditions per temperature (3) and per heat flux (4-6). More clearly, boundary conditions due to the prescribed heat flux are given by the expression (4). Further, boundary conditions due to the convection are given by the expression (5), where h_c is the convective coefficient and T_a is the temperature of the surrounding medium. Finally, boundary conditions due to the radiation (6) are not presently considered.

By the use of weak variational formulation, Fourier's law (2) and the use of the divergence theorem we obtain the linearised balance equation in the weak form (See [8]):

$$\int_{\Omega} \lambda \nabla T \cdot \nabla \Theta \, d\Omega = \int_{\Omega} f \cdot \Theta \, d\Omega \quad (7)$$

Further, using the primal finite element method [1,2] we end with the large scale system of linear equations in the matrix form:

$$[\mathbf{K}]\{\mathbf{T}\} = \{\mathbf{F}\} \quad (8)$$

where entries in matrix $[\mathbf{K}]$ and vector $\{\mathbf{F}\}$ are given by:

$$\mathbf{K}_{LM} = \sum_e \int_{\Omega_e} P_{L,a} \lambda P_{M,a} \delta\Omega_e \quad \mathbf{F}_M = \sum_e \int_{\Omega_e} P_M f \, d\Omega_e \quad (9)$$

On the other hand, we may use the original mixed finite element method [4] also, where we have large scale system of linear equations in the matrix form:

$$\begin{bmatrix} \mathbf{A}_{vv} & \mathbf{B}_{vv}^T \\ \mathbf{B}_{vv} & -\mathbf{D}_{vv} \end{bmatrix} \begin{bmatrix} \mathbf{q}_v \\ \mathbf{T}_v \end{bmatrix} = \begin{bmatrix} -\mathbf{A}_{vp} & -\mathbf{B}_{vp}^T \\ -\mathbf{B}_{vp} & \mathbf{D}_{vp} \end{bmatrix} \begin{bmatrix} \mathbf{q}_p \\ \mathbf{T}_p \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{F}_p + \mathbf{H}_p - \mathbf{K}_p \end{bmatrix} \quad (10)$$

The members of the entry matrices \mathbf{A} , \mathbf{B} and \mathbf{D} , and the column matrices \mathbf{F} , \mathbf{H} , and \mathbf{K} in (8), are respectively

$$\begin{aligned} \mathbf{A}_{LpMr} &= \sum_e \int_{\Omega_e} \mathbf{g}_{(L)p}^a V_L \mathbf{k}_{ab}^{-1} \mathbf{g}_{(M)r}^b V_M \, d\Omega_e; & \mathbf{B}_{LpM} &= \sum_e \int_{\Omega_e} \mathbf{g}_{(L)p}^a V_L P_{M,a} \, d\Omega_e \\ \mathbf{D}_{LM} &= \sum_e \int_{\partial\Omega_{ce}} h_c P_L P_M \delta\Omega_{ce}; & \mathbf{F}_M &= \sum_e \int_{\Omega_e} P_M f \, d\Omega_e \\ \mathbf{H}_M &= \sum_e \int_{\partial\Omega_{he}} P_M h \, d\Omega_{he}; & \mathbf{K}_M &= \sum_e \int_{\partial\Omega_{ce}} P_M h_c T_0 \, d\Omega_{ce} \end{aligned} \quad (11)$$

In upper expressions, T^L and \mathbf{q}^L are nodal values of the temperature scalar T and flux vector \mathbf{q} , respectively, P_L and V_L are corresponding values of the interpolation (local base) functions, $g_{(L)p}^a$ are the Euclidian shifting operators, Ω_i is the body of an element, h_c is the convective coefficient, h is the heat source, and T_a is the temperature of the surrounding medium, furthermore, $P_{M,a} \equiv \frac{\partial P_M}{\partial \xi^a}$.

In the present investigation, both finite element techniques, primal and mixed, will be used for validation of results in comparison to the theoretical ones given in the standards [9] and [10].

3. Numerical examples

Two numerical examples are given. First example is given by the motive to prove the usefulness of finite element codes in the calculation of the heat losses of the inhomogeneous building envelopes, and the second one with motive to show the use of finite element codes in the heat flow calculation of the complex three-dimensional analysis of buildings. The SI system of units is used throughout the text.

3.1 Heat flow through inhomogeneous wall

In the present example, the dependence of heat transfer coefficient U (U -value) of the building envelope's material properties is investigated. The inhomogeneous wall [9] build of light expanded clay aggregate block filled with polystyrene, shown in Fig.1, with or without mineral wool is considered. The polystyrene decreases the heat flow through the block. The aim of this example is to show that numerical simulation ensures fast, cheap and precise optimization of the building envelope from the cost / energy efficiency point of view.

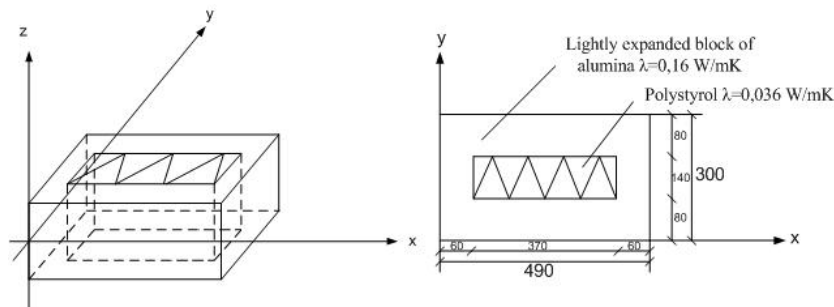


Figure 1 - Porous block of alumina filled with polystyrene

The wall made of these blocks and its representative part, which is investigated in this example, are shown in Fig 2. Representative blocks, are placed closely one to each other and connected with relatively thick horizontal layer of mortar, while there are no vertical layers of mortar. Therefore, heat is transferred mostly along axis y .

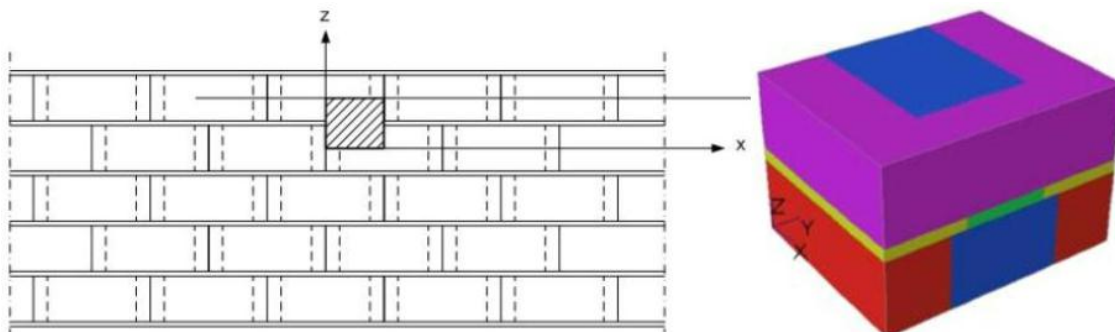


Figure 2 – Wall made of porous blocks filled with polystyrol, and representative part

Mortar between the blocks represents thermal bridge, which may be reduced by placing the mineral wool strip into it (see Fig. 3). It is for expected that the value of the overall U -value should be reduced by setting the mineral wool into the mortar, and by that, heat loss of this test model should be reduced. Calculation was made for the shadowed part shown in Figure 2.

There are planes of symmetry on $x = 0$ and $x = 245 \text{ mm}$, and $z = 0$ and $z = 200 \text{ mm}$, shown in Fig. 3 and 4, respectively. Boundary conditions in these planes are adiabatic. Temperatures on these boundaries, are set to be $T_{y=300} = 0 \text{ K}$, and $T_{y=0} = 1 \text{ K}$, as shown in Fig. 4. Complex process of convective heat transfer on the outdoor and indoor side of the inhomogeneous wall is expressed by coefficient of heat resistance $R_{y=0} = 0.04$ on the outdoor side and $R_{y=300} = 0.13$ on the indoor side of the wall (Figures 3 and 4). European standard CEN, made in 1996 [9], in which solution is obtained by the use of the simplified equations, gives in this case value $U_{\text{CEN}} = 0.355$ for model without mineral wool, as well as the value $U_{\text{CEN}} = 0.296$ for model with mineral wool.

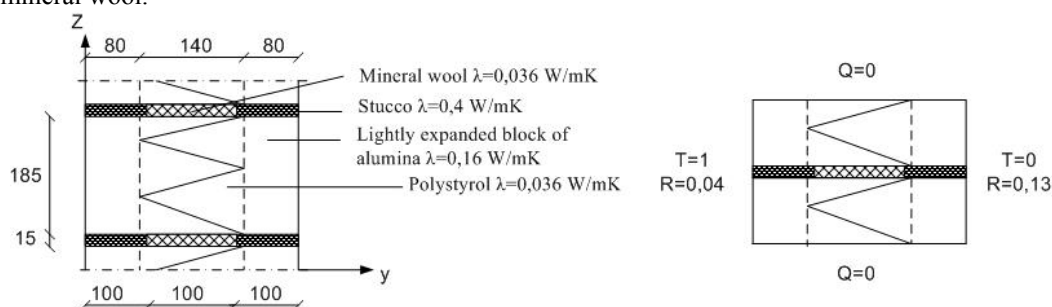


Figure 3 – Characteristic intersection of the wall in Y-Z plane

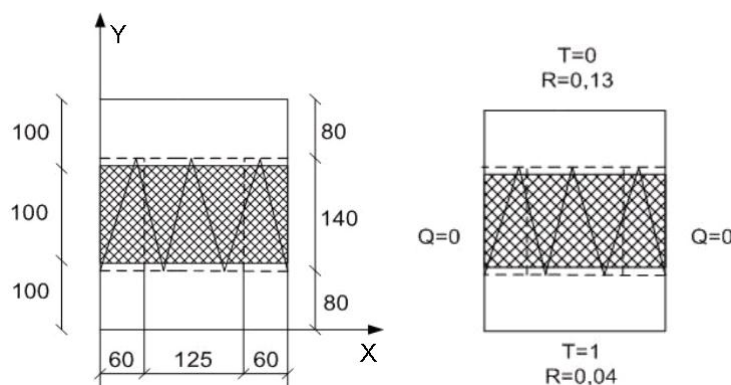


Figure 4 – Characteristic intersection of the wall in X-Y plane

Prescribed values of thermal resistance are encountered in the overall U -value by the next expression:

$$U = \frac{1}{(R_{y=0} + R_{y=300}) + R_{\lambda}}, \quad \text{where } R_{\lambda} = \frac{1}{U_{\lambda}} \quad \text{and} \quad U_{\lambda} = \frac{Q_{\lambda}}{A_{x-z}(T_{y=0} - T_{y=300})} \quad (12)$$

where Q_{λ} represents value of the heat flow on the indoor (or outdoor) side of the wall with area $A_{x-z} = 0.049$ (in X-Z plane).

In Table 1, U -value for the wall model without mineral wool, as well as execution times, for three consequently refined finite element meshes obtained by the software package “Straus7” [2], are given:

Number of elements	U [W/(m ² K)]	E [%]	Calculation time [s]
300	0.3704	4.33	0.5
480	0.3699	4.19	1.0
3840	0.3686	3.82	5.0
Theoretical solution [16]	0.3550		

Table 1 – U -values obtained using software package “Straus7” for wall model without mineral wool

Further, U -values for the model with mineral wool stripe obtained also by the software package “Straus7”, are given in Table 2.

Number of elements	U [$W/(m^2K)$]	ε [%]	Calculation time [s]
300	0.3065	3.55	0.5
480	0.3056	3.24	1.0
3840	0.3035	2.53	5.0
Theoretical solution [16]	0.2960		

Table 2 – U -values obtained using software package “Straus7” for wall model with mineral wool

From the given results we may conclude that obtained results converge without oscillations with the mesh refinement, which proves its high quality. Moreover, software package “Straus7” is specialized for some other types of structural analysis (thermoelastic, seismic,...) also, which increases its interoperability [10].

Nevertheless, heat flux field is what is of greater interest in this example, In Figure 5, heat flux component q^y for model with 3840 elements, calculated using the software package “Straus7”, is shown. In this figures we may also see good agreement of the results with theoretical solution from CEN [9].

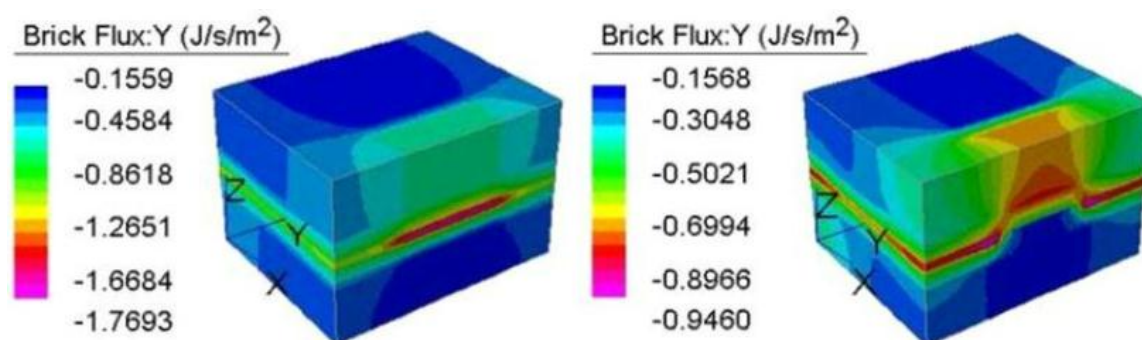


Figure 5 – Flux fields for representative part of inhomogeneous wall with and without mineral wool strip

From the results obtained, we may conclude that polystyrene inside the block, as well as mineral wool, significantly decreases the heat bridge effect.

Overall coefficient of heat transfer value calculated in software “Straus7” for superfine mesh of 3840 finite elements decreases from $U = 0.3686$ to $U = 0.3035$, which is relative decrease of 21.45%, while European standard CEN [9] for the same example, but based on the hand calculations, gives relative decrease of 19.93%.

It is very important to calculate how implementation of the mineral wool strip in order to diminish the heat loss, increases the price. Price of one block (with polystyrene) is 1.6 Eur, further mortar, along one block surface, without mineral wool costs 0.2 Eur, while mortar along one block surface with mineral wool costs 0.4 Eur. This means that the price of the standard wall of $3.18 \times 2.6 m^2$ area will be increased from 150 Eur to 165 Eur, i.e. 11.12%. From the upper analysis of the U -value calculation using the software package “Straus7”, follows that by increasing the wall price for 11.12%, we decrease heat loss for even 21.45%. In the heating season, for this standard wall example, heat loss should be decreased from 288 kWh/year to 237,13 kWh/year.

If ΔC is cost difference, where C_j is unit cost of thermal energy in Euros, and ΔQ heat loss difference, then from the next equation [5], we may calculate amortization period τ of the investments in the more expensive wall with mineral wool:

$$\tau = \frac{\Delta C}{C_j \cdot \Delta Q} = 5 \text{ years} \quad (17)$$

It should be emphasized that presently we intervene only in heat characteristics of the horizontal sections of the wall. Intervention on the outdoor section of the wall (building envelope) could even more diminish the heat losses. Therefore, the future work is oriented with numerical experiments of the building envelope that is coat by the silicone resin emulsion paints.

3.2 Heat flow through a corner

This example deals with heat flow through a corner and a single floor. The source of this benchmark in three-dimensional heat conduction problem is EN ISO 10211-1:1995 Case 3 [11], where heat flows and surface temperatures are given for the certain number of surfaces and points of a model, respectively.

An important point about this benchmark is that it demonstrates that software package “Straus7”, and alternatively in-house software package “FEMIX”, can be used to determine heat flows (gains/losses) throughout the structures. This can be very important for estimating energy requirements in buildings and other structures or processes. Material and geometry data are shown in Figure 6. The system of units used is SI, except that temperature is expressed in Celsius.

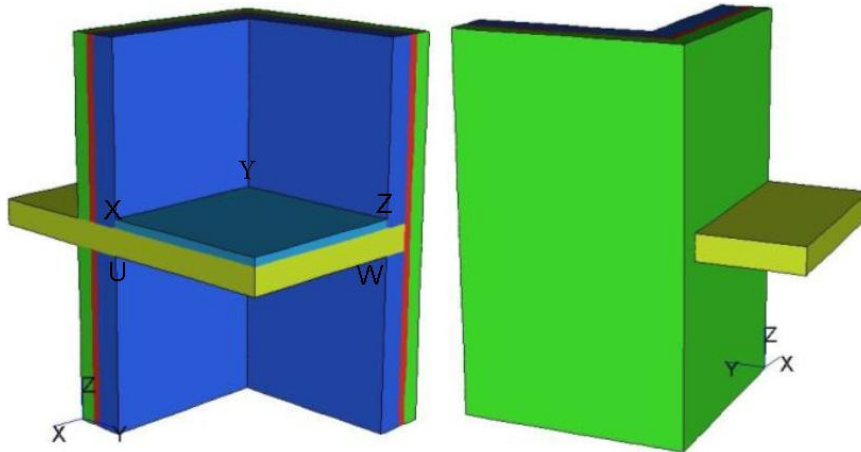


Figure 6. 3D Steady heat transfer problem of heat flow through a corner

Results of interest are the temperatures at the six points labeled U to Z in Figure 6 and the heat loss/gain through α , β and γ surfaces, shown in Figure 7. The materials are labeled with numbers 1-5. The boundary conditions applied are labeled with Greek letters α , β and γ (see Table 3 and Figure 7).

Material	λ	boundary condition	T_a	R
1	0.7	α	20	0.2
2	0.04	β	15	0.2
3	1.0	γ	0	0.05
4	2.5	δ	$Q = 0$ (adiabatic)	
5	1.0			

Table 3. Thermal conductivities and boundary conditions.

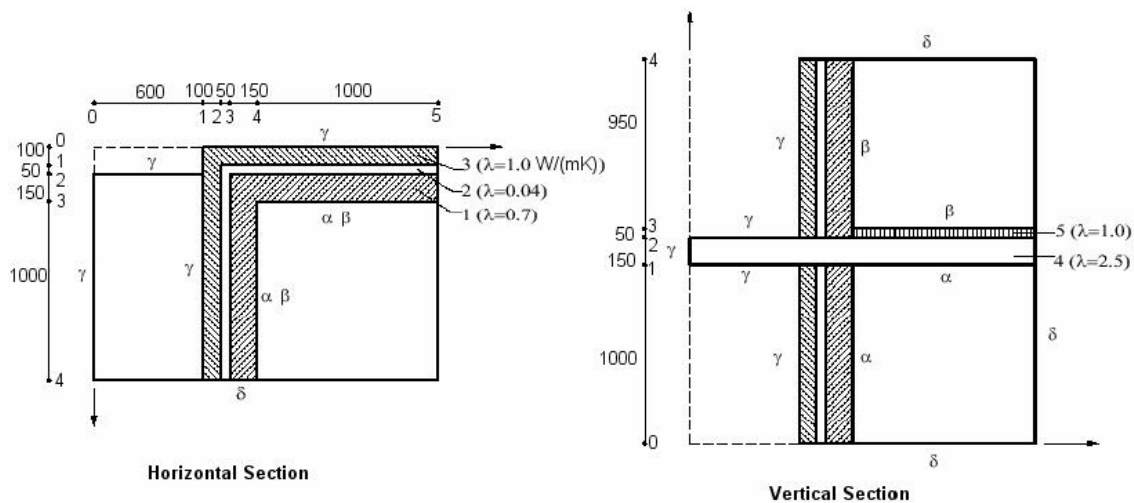


Figure 7: Horizontal and vertical sections showing geometry, material properties and boundary conditions

In the present example we will see that results obtained by the in-house finite element software package “FEMIX” based on the mixed finite elements “HC8/9” are comparable with the results obtained by the commercial software package “Straus7” based on the primal finite elements “HEXA8”. Therefore, we may say that both procedures are validated to each other.

The building corner was modeled using hexahedral brick elements. The number of finite elements employed in the present model is 1096. The results are summarized in Tables 4 and 5.

Element	Temperature Results $T [^{\circ}C]$					
	U	V	W	X	Y	Z
HEXA8	13.0	11.6	16.6	12.5	11.3	15.3
HC8/9	13.3	12.0	16.6	12.6	11.5	15.3
Target	12.9	11.3	16.4	12.6	11.1	15.3

Table 4. Summary of the temperature results

Element	Heat Flow Results $Q [W]$		
	α	β	γ
HEXA8	-47.20	-14.20	61.30
Target	-46.30	-14.00	60.30

Table 5. Summary of the heat flow results

The results given in Tables 4 and 5, indicate that total heat flows can be modeled quite well even with low order finite elements. Further, in Figures 8 and 9, temperature and heat flux field, calculated using the software package “Straus7” and in-house software package “FEMIX HC8/9” [3,4], are shown.

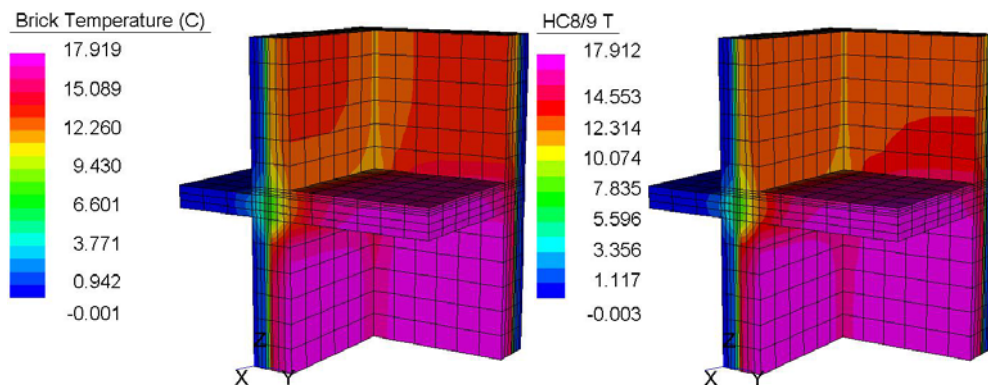


Figure 8 – Temperature contours obtained by “Straus7” and “FEMIX”, respectively

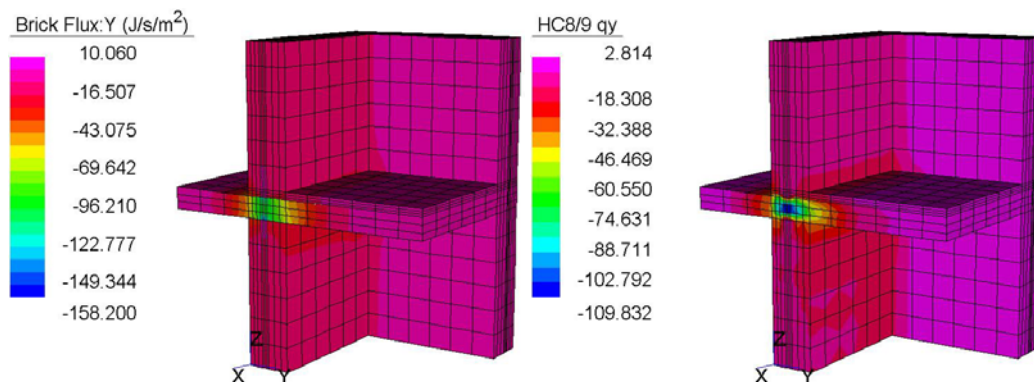


Figure 9 – Heat flux component field obtained by “Straus7” and “FEMIX”, respectively

From the results obtained, we may see that with a computationally inexpensive mesh (execution time is seven second for the “Straus7”) we successfully meet the target results.

4. Conclusions

The use of the finite elements method in the energy efficiency of buildings, is demonstrated. It is shown that described method may be treated as cheap, more efficient and more precise in regard to standard calculating methods. Moreover, it is shown that this method gives wide opportunities in the design process of the buildings and prediction of its thermal behavior. By simple introduction of the input characteristics of material, geometry and boundary conditions, the model problem is analyzed and optimized with possibility of three-dimensional visualization of the calculated temperature and heat flux. By the use of suggested simulation method in the design process of the building's envelope and HVAC system, optimization of the cost/energy efficiency rate is enabled.

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